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Caustic Interpretation and Engineering of the Synthesized Vortex Beams

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Abstract: We propose an effective scheme to interpret and engineer the caustics of the synthesized vortex beams which would be beneficial to a broad range of applications such as particle trapping and micromachining.

1. Introduction

Optical vortices (OVs) are structured light fields with helical phase fronts described as exp ($il\theta$), where l is the topological charge. Due to the doughnut-shaped spatial profile and phase singularity, the OVs have attract widespread interest. Recently, by superimposing the helical phase on different host beams (for example, Gaussian beams, Bessel beams and autofocusing beams), a series of axially symmetric vortex beams are synthesized and investigated [1-4]. In addition, there has been an increasing interest in the applications of vortex beams with different geometries, such as the rapid fabrication of micro-tubes [5]. For these applications, light tubes with predetermined geometries are required to be engineered. Here, we propose an analytical scheme to engineer the caustic geometries of the axially symmetric vortex beams. In this scheme, we deduce a set of formulas to describe the *general* caustic trajectories of the synthesized vortex beams based on the ray optics. The validity of our scheme is demonstrated by the agreement between the analytical, the numerical and the experimental results.

2. Theory and results

By combing the helical phase on the radially symmetric power-law phase of different host beams, the total phase of the synthesized vortex beams can read as:

$$\phi(r,\theta)=\varphi_{host}(r)+\varphi_{\text{vortex}}(\theta)=\varphi_{host}(r)+l\theta \tag{1}$$

In the classical geometric optics, light is assumed to propagate along rays and light field can be interpreted as the ensemble of families of rays emerging from any cross section along propagation. By defining a set of parameters as: as: $N\equiv\sqrt{l^2/r^2+[\phi'(r)]^2}/k$, $V\equiv\sqrt{1/N^2-1}$, $R\equiv l/kN$, $L\equiv RV$, $z_w=-r\phi'(r)L/l$ (the prime means the derivative with respect to r), we found that each bunch of straight rays emerging from the ring with a given radius r in the initial plane $z=0$ lie on a specific hyperboloid, defined by $\rho^2/R^2-[z-z_w]^2/L^2=1$ with $\rho^2=x^2+y^2$. Fig. 1(a) presents two bundles of such straight rays with different r, winding on two different hyperboloids. Therefore, the whole light field can be represented as the superposition of different hyperboloids.

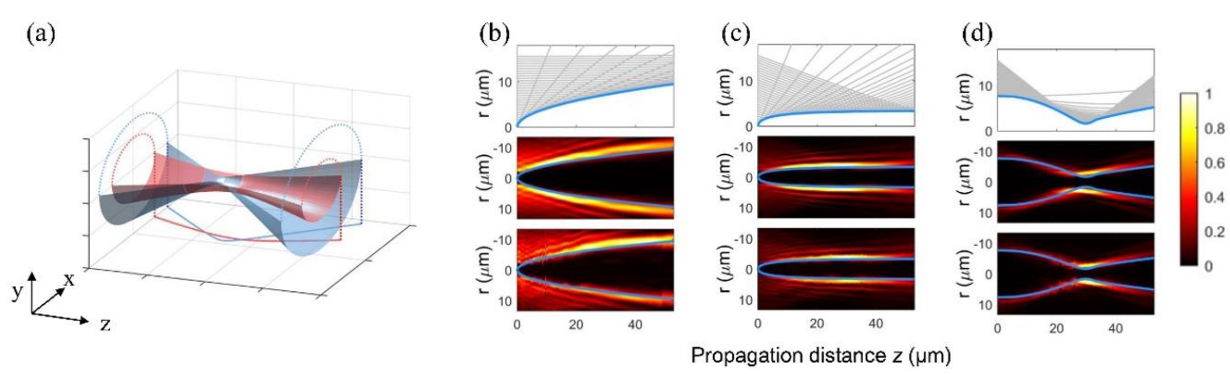

Fig. 1. (a) The x-z cut of caustic image for any synthesized axially symmetric vortex beam: two hyperboloids with different r. (b)-(d) The longitudinal image of caustics, together with the corresponding numerical and experimental results for different synthesized vortex beams with l=5: (b) vortex beam; (c) Bessel vortex beam; (d) abruptly autofocusing vortex beam. Blue solid lines indicate the corresponding analytical caustic trajectories obtained by solving Eq. (2).

In geometrical optics, the caustic is defined as the envelope of a family of curves or surfaces. With the help of the above mathematics, we found that the characteristic points forming the global caustic are the solutions of the following quadratic equation:

$$A\left(z-z_w\right)^2+B\left(z-z_w\right)+C=0$$
$$A=\left(LVN\right)^{-2}\frac{R'}{R},\quad B=\frac{z_w{}'}{L^2},\quad C=-\frac{R'}{R},\quad \Delta\equiv\sqrt{B^2-4AC} \tag{2}$$

which can be readily solved analytically. We emphasize that the set of solutions are the *general* formula describing the global caustic of any axially symmetric vortex beams synthesized from a given host beam. This is also demonstrated by their excellent agreement with both the numerical and the experimental results, as shown in Fig. 1(b)-(d). Furthermore, our analytical results can also be used to tailor the propagation features. Under a precondition $r \gg |l/\phi'|$, the global caustic, i.e. the solutions of Eq. (2), can be well approximated as:

$$z_1=-r\sqrt{k^2-\left[\phi'(r)\right]^2}\Big/\phi'(r),\quad ,\ \text{and/or}\quad z_2=-k\left(\sqrt{1-\left[\phi'(r)/k\right]^2}\right)^3\Big/\phi''(r) \tag{3}$$

In this way, the caustics $z(r)$ can be tailored by engineering the host phase in Eq. (1) with a target profile $c(z) = r(z)$. The numerical and experimental results of two engineered axially symmetric vortex beams are shown in Fig. 2. The corresponding retrieved caustic profiles are in excellent agreement with the pre-determined target geometries.

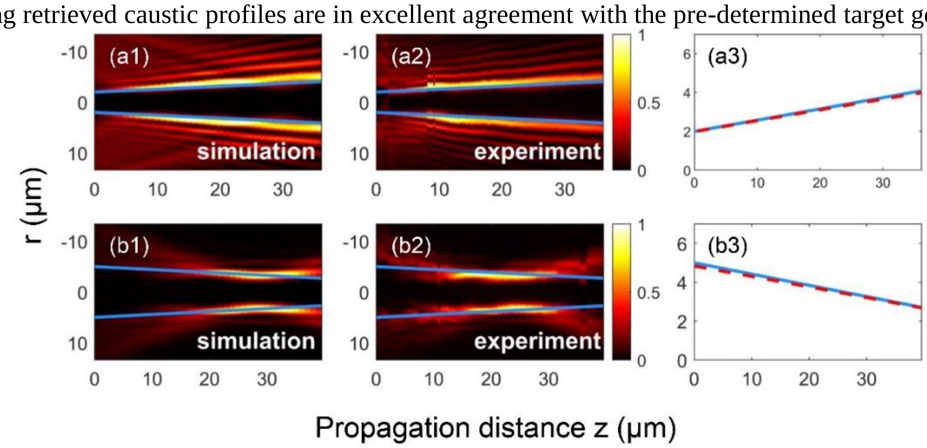

Fig. 2. Numerical and experimental results of two vortex beams with engineered caustics along linear ramp profiles: (a1) and (a2), $c(z)=a+bz$ with a=2μm, b=0.058; (b1) and (b2), $c(z)=a-bz$ with a=5μm, b=0.058; The predetermined caustic trajectories (blue solid lines) are superimposed on the x-y cut of the intensity profiles. (a3) and (b3), comparison of the target profiles (blue solid lines) and the retrieved profiles (red dashed lines) calculated from Eq. (2) and Eq. (3).

3. Conclusion

In conclusion, we obtain a set of analytical formulas to describe the caustics of the axially symmetric vortex beams. Based on several illustrative example synthesized vortex beams, the validity of our analytical results is proved. We have also shown the feasibility of engineering the vortex caustics in our scheme by the excellent agreement of the retrieved results with both numerical and experimental results. These results are highly promising in engineering and adapting the vortex beams to numerous applications, such as particle micromanipulation and material processing.

4. References

[1]. S. Orlov, K. Regelskis, V. Smilgevicius, and A. Stabinis, “Propagation of Bessel beams carrying optical vortices,” Opt. Commun. **209,** 155–165 (2002).

[2]. H. T. Dai, Y. J. Liu, D. Luo, and X. W. Sun, “Propagation dynamics of an optical vortex imposed on an Airy beam,” Opt. Lett. **35,** 4075–4077 (2010).

[3]. Y. Jiang, K. Huang, and X. Lu, “Propagation dynamics of abruptly autofocusing Airy beams with optical vortices,” Opt. Express **20,** 18579–18584 (2012).

[4]. M. Goutsoulas, D. Bongiovanni, D. Li, Z. Chen, and N. K. Efremidis, “Tunable self-similar Bessel-like beams of arbitrary order,” Opt. Lett. **45,** 1830-1833 (2020).

[5]. L. Yang, D. Qian, C. Xin, Z. Hu, S. Ji, D. Wu, Y. Hu, J. Li, W. Huang, and J. Chu, "Direct laser writing of complex microtubes using femtosecond vortex beams," Appl. Phys. Lett. **110,** 221103 (2017).