Engineering synthesized vortex beams

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Abstract: We propose an effective scheme to engineer the light tubes of vortex beams with predetermined geometries and controllable intensity profiles. This is beneficial to a broad range of applications such as particle trapping and micromachining.

1. Introduction

Optical vortices (OVs) are structured light fields incorporating helical phase factors described as exp ($il\theta$), where l is the topological charge. Recently, a series of axially symmetric vortex beams were synthesized and investigated by superimposing the helical phase on their original host phases [1-4]. Due to the phase singularity induced doughnut-shaped spatial profile, the vortex beams have attracted widespread interests for microscopy, laser ablation and fabrication of two-photon polymerizations [5]. But for all these applications, tailoring simultaneously the spatial profile and the intensity profile is a key problem. Here, we solve this issue by combining the properties of caustics and amplitude engineering. The validity of our scheme is demonstrated by the excellent agreements between the analytical, the numerical and the experimental results.

2. Theory and results

By combing the helical phase on the radially symmetric phase of different host beams, the total phase of the *synthesized* vortex beams, in the initial plane $z=0$, can read as:

$$\phi(r,\theta)=\varphi_{host}(r)+\varphi_{\text{vortex}}(\theta)=\varphi_{host}(r)+l\theta \tag{1}$$

In the classical geometric optics, light is assumed to propagate along rays and can be interpreted as the ensemble of families of rays emerging from any cross section along propagation. By defining a set of parameters as: $N \equiv \sqrt{l^2/r^2+\left[\phi'(r)\right]^2}\Big/k$, $V \equiv \sqrt{1/N^2-1}$, $R \equiv l/kN$, $L \equiv RV$, $z_w = -r\phi'(r)V/kN$ (the prime means the derivative with respect to r), we found that each bunch of straight rays emerging from the ring with a given radius r in the initial plane $z=0$ lie on a specific hyperboloid, defined by $\rho^2/R^2-\left[z-z_w\right]/L^2=1$ with $\rho^2=x^2+y^2$, as shown in Fig. 1(a). Therefore, the whole light field can be represented as the superposition of different hyperboloids.

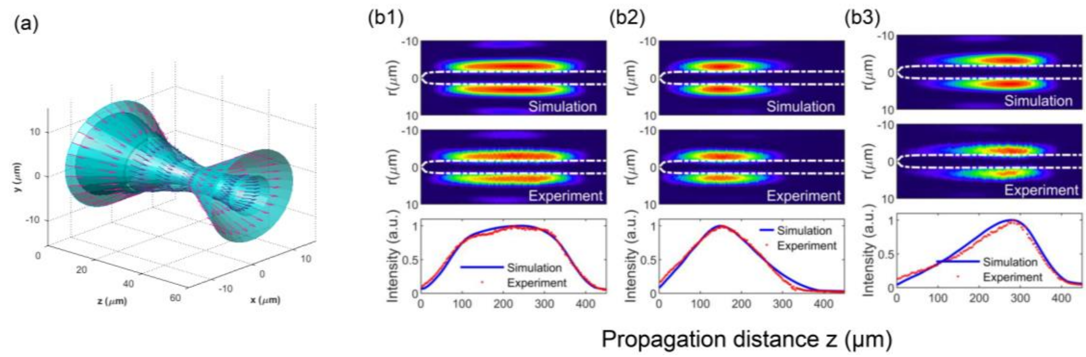

Fig. 1 (a) Schematic of the caustic interpretation of the vortex beam. Bessel vortex beams (cone angle γ=5°and l=1) with three intensity profiles of the tubular main lobes: (b1) a plateau with two parabolic edges, (b2) and (b3) uniformly increasing leading edge followed by a parabolic falling edge intensity with different locations of the maximum intensity. The analytical caustics obtained from our Eq. (2) are superimposed in the 2D intensity profiles.

As the caustic is defined as the envelope of a family of curves or surfaces, the global caustic [z, ρ (z)] can be readily solved from the solutions of the following quadratic equation:

$$(LVN)^{-2}\frac{R'}{R}(z-z_w)^2+\frac{z_w{}'}{L^2}(z-z_w)-\frac{R'}{R}=0 \tag{2}$$

We emphasize that the set of solutions to Eq. (2) is actually a *general* formula, describing the global caustic of the synthetic vortex beams with the phase expressed in Eq. (1). This is of primary importance because it allows us to straightforwardly link the phase and intensity in the *initial* plane with the intensity distribution around the main lobe of the beam at *any* further propagation distance.

Taking the Bessel vortex beam with cone angle γ=5°and l=1 as an example, the solutions are plotted in white dash-dotted lines in Fig. 1(b). The validity of our solutions is demonstrated from their excellent agreements with both the numerical and the experimental results. Besides, this excellent agreement of our solutions to Eq. (2) with the experiments can also be found in our previous work on the abruptly autofocusing vortex beams [4].

Practically, the inverse problem on *tailoring the caustic*, i.e. the tube shape, *by an engineered host phase* $\varphi_{host}(r)$ is more interesting. Under a precondition $r \gg -l/\phi'(r)$, the global tubular caustic can be well approximated as:

$$z_1 = -r\left(k^2 - \left[\varphi'_{host}(r)\right]^2\right)^{1/2} \Big/ \varphi'_{host}(r) \text{ and/or } z_2 = -k\left(1 - \left[\varphi'_{host}(r)/k\right]^2\right)^{3/2} \Big/ \varphi''_{host}(r) \quad (3)$$

In this way, a target profile of the tubular shape $\rho(z)$ = $c(z)$ could be engineered by solving the host phase $\varphi_{host}(r)$ from $z_{caustic}(r) = c^{-1}(z)$. The numerical and experimental results of two synthesized vortex beams with opposite tapered tubes are shown in Fig. 2. The engineered tubular shapes show excellent agreement with the pre-determined target profiles.

With the help of a programmable spatial light modulator, we can further engineer the intensity profiles of the main tubular lobes of Bessel-like vortex beams by tailoring the input amplitude [3,6]. As for this, we design three intensity profiles for the Bessel vortex beam and two intensity profiles for the above synthesized tapered vortex beams. It is obvious that the engineered shape and intensity profiles of the main tubular lobes are both in excellent agreement with the pre-determined target beams as shown in Fig. 1 and Fig. 2.

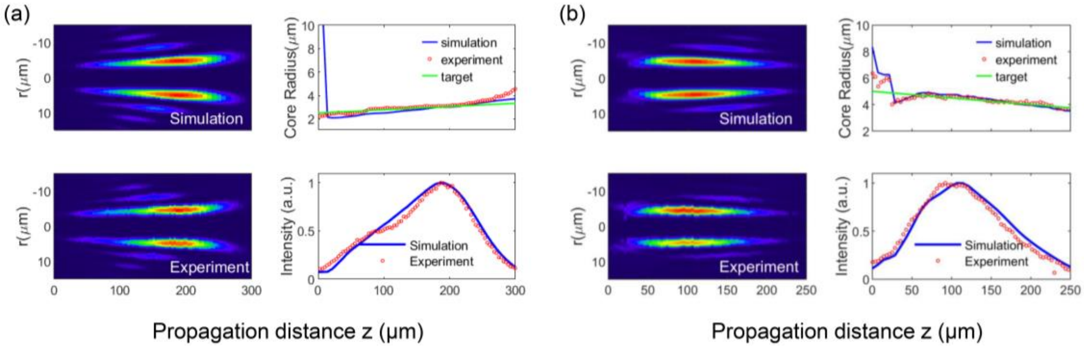

Fig. 2. The numerical simulation results and experimental results of two particular examples with opposite tapered tubes: (a) $c(z)=a+bz$ with a=2.5, b=0.0028, (b) $c(z)=a-bz$ with a=5, b=0.005. The topological charge is l=3.

3. Conclusion

In conclusion, we obtain a set of analytical formulae to describe the global caustics of the axially symmetric vortex beams. As a demonstration, the caustics of Bessel vortex beams predicted from our solution show excellent agreement with both the simulations and the experiments. Besides, our analytical results can be further extended to tailor the features of the synthesized vortex beams with both pre-determined tube shape and intensity profiles. The feasibility of our engineering method is demonstrated by the excellent agreement between our analytical, the numerical and the experimental results. These results are highly promising in adapting the family of vortex beams to numerous applications, such as particle micromanipulation and material processing.

4. References

[1]. S. Orlov, K. Regelskis, V. Smilgevicius, and A. Stabinis, “Propagation of Bessel beams carrying optical vortices,” Opt. Commun. **209**(1-3), 155–165 (2002).

[2]. H. T. Dai, Y. J. Liu, D. Luo, and X. W. Sun, “Propagation dynamics of an optical vortex imposed on an Airy beam,” Opt. Lett. **35**(23), 4075–4077 (2010).

[3]. M. Goutsoulas, D. Bongiovanni, D. Li, Z. Chen, and N. K. Efremidis, “Tunable self-similar Bessel-like beams of arbitrary order,” Opt. Lett. **45**, 1830-1833 (2020).

[4]. N. Xiao, C. Xie, E. Jia, J. Li, R. Giust, F. Courvoisier, and M. Hu, “Caustic interpretation of the abruptly autofocusing vortex beams,” Opt. Express **29**(13), 19975–19984 (2021).

[5]. L. Yang, D. Qian, C. Xin, Z. Hu, S. Ji, D. Wu, Y. Hu, J. Li, W. Huang, and J. Chu, “Direct laser writing of complex microtubes using femtosecond vortex beams,” Applied Physics Letters **110**(22), 221103 (2017).

[6]. I. Ouadghiri-Idrissi, R. Giust, L. Froehly, M. Jacquot, L. Furfaro, J. M. Dudley, and F. Courvoisier, “Arbitrary shaping of on-axis amplitude of femtosecond Bessel beams with a single phase-only spatial light modulator,” Opt. Express **24**, 11495-11504 (2016).